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TEACHING MATHEMATICS USING EVERYDAY CONTEXTS: WHAT IF ACADEMIC MATHEMATICS IS LOST?

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Mathematics education researchers argue that mathematics should be taught using everyday contexts so that the learning of mathematics can be meaningful to students. Although the learning of mathematics through everyday contexts is interesting for most students, many of them cannot make a leap from these contexts to academic mathematics. Because of this difficulty, teachers need to make a deliberate attempt to help students connect everyday and academic mathematics.

Introduction

During the past two decades, mathematics education researchers, who are especially interested in ethnomathematics, have explored the relationship between mathematics in and out of school (D'Ambrosio, 1985; Gerdes, 1996; Nunes, 1992). Out of school mathematics is usually carried out in everyday setting, which is very different from an academic setting of schools (Carraher, Carraher, & Schliemann, 1985; Saxe, 1991). While academic mathematics is still viewed as a culture and context free discipline, mathematics in everyday settings is determined by socio-cultural backgrounds of students. Ethnomathematicians, however, believe that mathematics both in and out of school must be based on socio-cultural practices of students. According to Gerdes, ethnomathematics researchers "emphasize and analyze the influences of socio-cultural factors on the teaching, learning and development of mathematics." (p. 917).

The psychology of mathematics education has been deeply influenced by the findings of the research carried out from a socio-cultural perspective. In the 1998 annual meeting of the International Group for the Psychology of Mathematics Education, one major theme of research was focused on mathematics in and out of school. The researchers in the meeting argued that the learning of mathematics becomes meaningful to students if their own cultural contexts are used in mathematics classrooms (Civil, 1998; Presmeg, 1998). If mathematical concepts, ideas, and skills are developed through students' everyday contexts, they may be more motivated to learn and develop a better understanding of mathematics.

The task of connecting students' everyday contexts to academic mathematics is not easy (Lave, 1988; Saxe, 1991; Walkerdine, 1990). These educators argue that students construct their everyday experiences in contexts different from the school context, and transferring ideas from one context to another is hard because the emergent goals are different. This may explain the discrepancy between school and out-of-school mathematics experiences reported by numerous educators (Carraher, Carraher, & Schliemann, 1985; D'Ambrosio, 1985; Nunes, 1992; Saxe, 1991).

Although the above researchers have shown a discrepancy between school and out-of-school mathematics, their main focus was not to see whether students' mathematical abilities could be enhanced when taught using their own everyday contexts. Nor was their focus to investigate students' feelings and interests about mathematics when taught through such contexts. Instead, these researchers focused on how students' understanding of mathematics is embedded in their culture and personal experiences. More research is needed to demonstrate how students can be helped to learn academic mathematics using students' socio-cultural contexts. In this paper, I examine the influence of everyday social contexts in the teaching of mathematics to future elementary school teachers.

Methodology of the study

The data for this study were collected from a group of preservice teachers enrolled in a course entitled "Number Systems" taught by this investigator at Eastern Connecticut State University. The majority of the preservice teachers in this group did not have a sound mathematical background. Only about 10% the preservice teachers had taken some advanced mathematics courses such as calculus. For the majority of the preservice teachers this was their first college mathematics course. These preservice teachers did not have a good experience of learning mathematics in schools. They feared and even hated mathematics.

The purpose of the course was to teach academic mathematics using students' everyday contexts as far as possible. All students were required to keep journals throughout these courses describing their mathematical understandings and feelings. Students were evaluated based on various quizzes, class presentations, journals, midterm and final examinations. In all these evaluations, they were required to demonstrate their understanding of academic mathematics. Although various types of problems were asked to the preservice teachers the following problem called a "shopping problem" was used to evaluate preservice teachers' ability to understand academic mathematics using their everyday cultural contexts. The problem was modified from one of the textbooks usually used in the "Number Systems" course and represents the types of everyday socio-cultural contexts used in the class.

> Two friends are shopping together when they encounter a special "3 for 2" shoe sale. If they purchase two pairs of shoes at the regular price, a third pair (of lower or equal value) will be free. Neither friend wants three pairs of shoes, but Pat would like to buy a \$56 and a \$39 pair while Chris is interested in a \$45 pair. If they buy the shoes together to take advantage of the sale, what is the fairest share for each to pay? (Adapted from Musser & Burger, 1997, p. 15)

The above problem was asked as a pilot problem to a group of preservice teachers in the previous year. The majority of the preservice teachers in that year did not provide a mathematical response to this problem. Their responses varied from "since Pat and Chris are friends they could divide the saving in any way they wanted," "I would not worry about the split but treat ourselves with a good lunch," to "just split the savings of \$39 evenly". Because of these general responses obtained from preservice teachers in the first year, preservice teachers in the succeeding year were specifically asked to provide their *mathematical reasoning* to the problem. All 32 preservice teachers in the class wrote their responses in the blank sheet of paper provided by the investigator.

After the analysis of the written responses, a total of six preservice teachers were interviewed to explore more about their solutions. The selection of preservice teachers was purposive in order to include preservice teachers of various ability levels. Since the interviews were conducted after the completion of the course, final grades of the students were used to determine their ability levels. One preservice teacher was a high achiever who obtained *A* in the course. Four were middle achievers, who got *B*'s in the course. One was a low achiever, with a *C*. Each interview lasted approximately 20-30 minutes. During the interviews, the preservice teachers were shown their written work and asked why they chose their responses. They were told that they could change their responses if they wanted.

Data Analysis Procedures

Preservice teachers' written responses to the problem were categorized, coded, and tabulated to determine their frequencies. Each interview tape was first audiotaped and then transcribed. Preservice teachers' strategies of solving the problem were determined by analyzing their written responses. The responses to the interview were used to determine the reasons why preservice teachers chose their solutions in the written task. Interview transcripts were also used to determine whether or not preservice teachers were consistent in their thinking.

Results and Discussion

All 32 preservice teachers in the class agreed that they commonly encounter these kinds of sales in their everyday life in the United States. However, the majority of them did not provide an appropriate academic solution to the problem. Since they had studied ratio, proportion, and percent in the class, an appropriate mathematical solution to this problem would have been to use a method to determine the amount of savings to Pat and Chris on a proportional basis. The following two solutions are considered appropriate based on the teaching in the classroom:

(i) The total cost of three pairs of shoes is \$56+\$45+\$39=\$140. The cost for Pat is \$56+\$39=\$95 and the cost for Chris is \$45. Since there is a saving

(ii) of \$39 out of \$140, Pat should save $39\times\frac{95}{140}=\$26.46$ and Chris should save $39\times\frac{45}{140}=\$12.54$. So Pat should pay \$95-\$26.46=\$68.54 and Chris should pay \$45-\$12.54=\$32.46.

(iii) Pat and Chris save a total of \$39 out of \$140. So their percent saving is $\frac{39}{140}\times100=27.85\%$. Hence, Pat should save 27.85% of \$95, which is \$26.46 and Chris should save 27.85% of \$45, which is \$12.54. So Pat should pay \$95-\$26.46=\$68.54 and Chris should pay \$45-\$12.54=\$32.46.

Not a single preservice teacher in the class gave one of the above two responses. Their solutions widely varied. Out of 32 respondents, eight said that "Pat should pay 2/3 of the price and Chris should pay 1/3. The total cost of the three pairs after the saving is \$56+\$45=\$101. Two thirds of \$101 is \$67 (rounded to the nearest dollar) and one third is \$34 (rounded to the nearest dollar). Hence Pat should pay \$67 and Chris should pay \$34." Here is a representative response from a preservice teacher:

> Chris and Pat would need to add up the cost of the two pairs of shoes that cost the most money because in a "3 for 2 sale" you pay the price of the two most expensive ones. The total of the two most expensive shoes would be \$56+\$45=\$101. Pat wants two pairs of shoes and Chris is only getting one pair, they need to divide \$101 by 3. The result when divided by 3 is \$33.66. Chris who is getting only one pair of shoes would pay 1/3 of the cost as \$33.66. Pat who is getting 2 pairs should pay 2/3 of the cost, which would be \$67.32. And everyone is happy. Chris saved \$11.34 off of the original \$45 and Pat saved \$27.68 off of the two pairs of shoes.

The above solution is clearly communicated. However, the shares are not distributed proportionally based on the cost of shoes.

Approximately the same proportion of the preservice teachers (seven out of 32) thought that both Pat and Chris should divide the savings of \$39 evenly. They said if there was no sale Pat would have paid \$56+\$39=\$95 and Chris would have paid \$45. Hence "Pat should pay \$95-\$19.50=\$75.50 and Chris should pay \$45-\$19.50=\$25.50." For example,

> The fairest thing would be to split the savings on the free pair in halves (\$39/2=\$19.50) and use the \$19.50 to subtract from the total price of the shoes they want. So the amount for Pat to pay is (\$56+\$39)-\$19.50=\$75.50 and for Chris to pay is \$45-\$19.50=\$25.50.

Nobody was worried that the sharing using the above method was unfavorable to Pat. He was saving only 20.5% of his original price of \$95 whereas

Chris was saving 43.3% of his price of \$45. When this was brought to their attention in the interviews, they argued that the sharing was still fair because both of them were willing to spend their original cost if there was no sale. Moreover according to these respondents they should not be arguing about how to split this money because both of them are friends.

Five preservice teachers decided to split the saving of \$39 into three parts and provide \$26 to Pat and \$13 to Chris. Hence for them the fairest shares would be that Pat pays \$69 and Chris pays \$32. Here is one response that exemplifies this method:

> Pat gets 2 pairs or 2/3 of the total shoes and Chris gets 1 pair or 1/3. Since the third (free) pair, which costs \$39, is free they should divide it by 3 and get \$13 per 1/3 of savings. Chris bought one pair so he should pay \$45-\$13=\$32. Since Pat has two pairs of shoes, he should pay \$95-\$26=\$69. Although Chris did not get a free pair he did save quite a bit of money.

The preservice teachers who split \$39 in three equal parts were mathematically fair than preservice teachers who simply split the money evenly. According to this new method Pat was saving 27.4% of the original price and Chris was saving 28.9% of the original \$45. The percentage was quite close because \$95 is only little bit more than the double of \$45. However if the difference between the cost of two pairs \$56 and \$45 was too high the percent savings would have been substantially different. This kind of complex proportional thinking was not demonstrated in any of the methods provided by the preservice teachers.

There were other preservice teachers who did not demonstrate any mathematical understandings. Three preservice teachers said that since both Pat and Chris were friends they could simply divide the total of \$101 evenly ~~and each pay~~ \$50.50. When asked why Chris should pay \$50.50 when the original price of the shoes he wanted to buy was only \$45, they could not give any mathematical reasoning and simply stated that if Chris is a really good friend then he should not mind paying extra \$5.50 for Pat especially because of a good bargaining opportunity. One of these preservice teachers said, "since Chris does not want three pairs of shoes she should be willing to pay the extra \$5.50 because of the bargain." Three other preservice teachers said that since Pat spent more money than Chris Pat should get the \$39 pair free. These preservice teachers were clearly not thinking mathematically. They were using friendship as a reason. It appeared that some of the preservice teachers, who had a weak mathematical background, simply wanted to avoid high level thinking required to solve this problem.

It is interesting to note that some preservice teachers who had a good mathematical understanding of the problem thought that dividing the saving evenly is fair. Ayaz was the most capable mathematics student in the class. He was the only student who got a perfect *A* in the class. He reasoned that the money should be split evenly between Pat and Chris. I provided a counter argument saying that the sharing between Pat and Chris can be considered mathematically fair only if their savings are proportional to their original costs. The participants in the study accepted my argument as an alternative to their solutions. However many of them were not willing to change their thinking. In the final interview Ayaz said that they can split the saving of $39 evenly. So Pat would pay $95-$19.50=$75.50 and Chris would pay $45-$19.50=$25.50. He stated that they can also split money proportionally such as $39 \times \frac{95}{140} = \$26.50$ for Pat and $39 \times \frac{45}{140} = \$12.50$ for Chris. Nevertheless, he insisted that splitting saving evenly was still fair. The following transcript between the researcher (Resh) and a preservice teacher (Ayaz) illustrates this issue:

Resh: Is splitting evenly a mathematical response or an everyday common sense kind of response?

Ayaz: I think it's a mathematical response. The common sense response would be spending more so that I should get a larger discount.

Resh: I was thinking that common sense response would be "Let's make it half-half." Why bother?

Ayaz: I can see that. It's an easier way to do. Nevertheless, Pat would be happy as long as she spends less than $95. I would not even bother to split the money. I would rather go out and treat ourselves with lunch. I think it's a philosophical and political question.

The above transcript indicates that Ayaz was comfortable with his earlier solution even though he understood how to split money proportionally. Other preservice teachers had a similar kind of argument: Debi, for example, determinined that Pat should pay $67.33 and Chris should pay $33.67. When asked if her method was fair, she argued that the method was fair from a real life perspective. Below is the excerpt from the interview with her:

Resh: Is that fair?

Debi: Pat is getting two pair of shoes and Chris is getting one pair of shoes. It should not matter. It is a split of free money. Doesn't matter how much money they are spending.

Resh: Don't you think their savings should be proportional to the amount of money they are spending?

Debi: May be from a mathematical perspective; but not from a shopping perspective, because they each have a choice of how much they want to spend. She happens to like $56 and $39 pair shoes. Well then that's what she should be willing to pay. If they were buying exactly the same pair of shoes, the splitting of money [on a proportional basis]

should have been considered. Since they have a choice they don't need to consider the split [proportionally].

Other preservice teachers such as Mona, Riva, Andy, and Hana were all satisfied with their way of solving this problem, which was not based on proportion of their cost prices. While preservice teachers' responses to the shopping problem were mostly based on the context of friendship, their subject matter knowledge of mathematics was not always academically strong. Many preservice teachers solved the shopping problem using a simplistic mathematical procedure even when the problem required a complex thinking. Their solutions did not involve complex mathematical thinking such as splitting the saving on a proportional basis. The majority of the students simply decided to split the saving based on the number of shoes purchased without considering their cost prices. No one in the class split the saving based on how much Pat and Chris would have spent if there were no sale. A few students did not demonstrate any mathematical understandings at all.

Conclusions

The results of this study indicate that the teaching of mathematics using students' everyday contexts does not necessarily enhance their understanding of academic mathematics. Instead of using academic mathematical concepts such as ratio, proportion, and percent, many preservice teachers solved the shopping problem based on a concept of simple division. When asked why they did not use a proportional reasoning the preservice teachers argued that they would not really set up a complex proportional procedure if they had to split the saving in a real life situation. They argued that since the problem appeared real they used a simple division, which many of them would actually use in their real lives.

The preservice teachers did have difficulty in using a proportional reasoning in this problem. Does it mean that we should avoid these kinds of problems in mathematics classrooms? If we do not use real life contexts like this, preservice teachers will see mathematics as a collection of isolated facts and skills to be memorized. It is therefore important to use these kinds of problems. Actually we need to use more of these problems and emphasize the fact that students are required to provide appropriate mathematical response to the problem based on what is taught in the class. In the shopping problem, for example, the use of simple division would have been fine if the responses were from elementary school students. However the responses to the problem from preservice teachers should include higher level mathematics of ratio, proportion, and percent. It appears that we must make our expectations clear to students and emphasize the fact that the purpose of using real life context in a mathematics class is to learn as much academic mathematics as possible. If this emphasis is not made students will simply bog down in contexts and not learn mathematics. Also, as Walkerdine (1990) argues, teachers should be aware that everyday practice of mathematics is discursively different from school practice and so the relation between everyday and school practices "is far more complex than is suggested by the notion of doing mathematical examples in familiar contexts" (p. 54).

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Learning mathematics in heterogeneous as opposed to homogeneous classes: Attitudes of students of high, intermediate and low mathematical competence.

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Abstract

Seventh- and eighth-grade students, who studied mathematics in heterogeneous settings organized for small group work according to a 'cooperative seating plan', were examined as to their attitudes to studying mathematics in heterogeneous classes as opposed to homogeneous classes. The eighth graders concurrently studied part of their mathematics curriculum in homogeneous classes. All students believed that studying in these groups facilitated their learning. Most students favored learning in heterogeneous classes. However, the eighth-grade, low achievers were ambivalent: they favored heterogeneous classes provided their assessment grades were higher. In response, an evaluation model is proposed to answer both the learning and psychological needs of students of diverse abilities studying in heterogeneous classes.

Research has cast doubt whether tracking is the correct way of dealing with diversity in abilities in the classroom. Not only has research shown that learning in low tracks significantly reduces achievement (e.g. Gamoran & Mare, 1989) but it has also been shown that 'top track' mathematics students can achieve as well in heterogeneous classes as in the tracked classes (e.g. Linchevski & Kutscher, 1998). Theorists claim that tracking is a central source of social inequity (e.g. Braddock, 1990). All this suggests that, whenever possible, mixed-ability classes should be the preferred learning setting in school. Many researchers argue for the value of cooperative learning in groups as a means of promoting attitude, motivation and achievement (e.g. Slavin, 1996) and for cognitive growth (e.g Webb (1989)). On the other hand Cobb suggests that small-group interaction is more productive when the interactions are multivocal and when the conceptual possibilities between the participants are relatively small (Cobb, 1996). This implies homogeneous grouping that "clashes with a variety of other agendas... including issues of equity and diversity" (ibid p.125).

This paper offers a 'cooperative-learning seating plan' that may reconcile these two seemingly contradictory approaches: cooperative-learning in small heterogeneous settings among participants whose cognitive capabilities are similar. This study examined a) the attitude of students who concurrently studied part of their mathematics curriculum in heterogeneous classes, where this cooperative-learning seating plan was adopted, and part of their mathematics curriculum in homogeneous settings and b) the attitudes of students who studied mathematics only in heterogeneous settings with this cooperative-learning seating plan. The conjecture was that all levels of students would prefer learning in these heterogeneous classes to learning in their homogeneous ones. These outcomes were expected since the heterogeneous learning environment was designed using the theoretical considerations and previous research results reported above.